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JACOBS SCHOOL OF ENGINEERING
Department of Mechanical and Aerospace Engineering

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Aircraft Design Optimization Using Reachability

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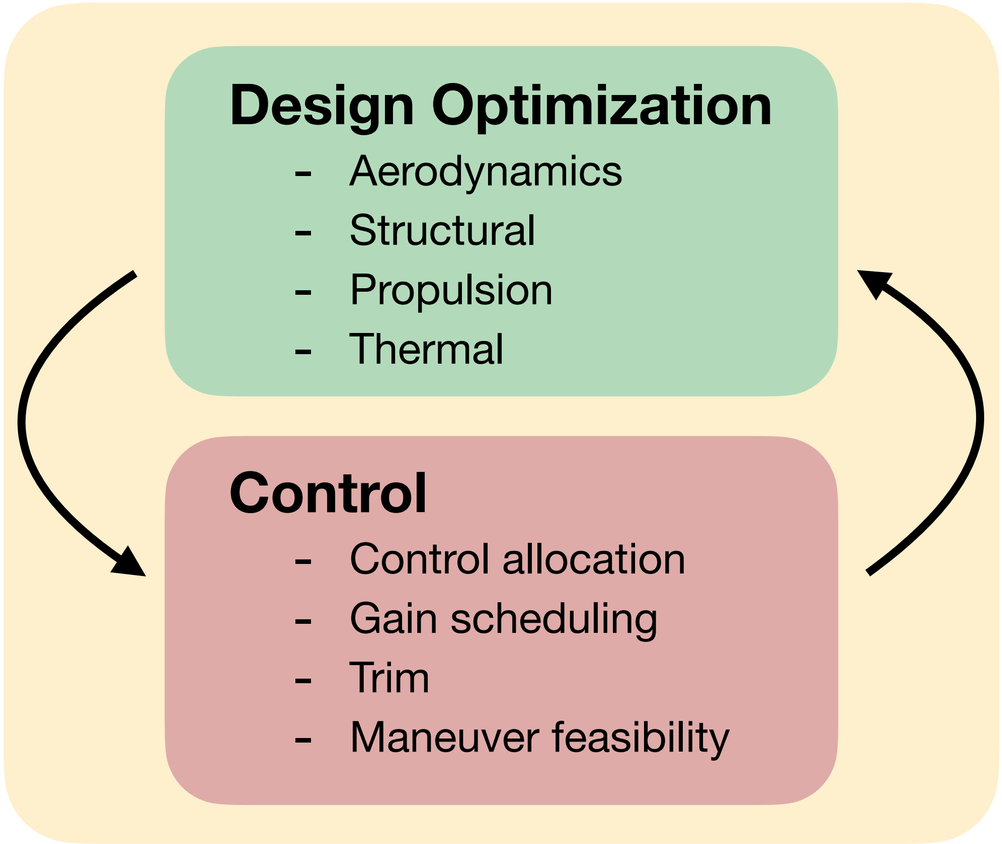
Traditional design of aircraft often separates control analysis from design



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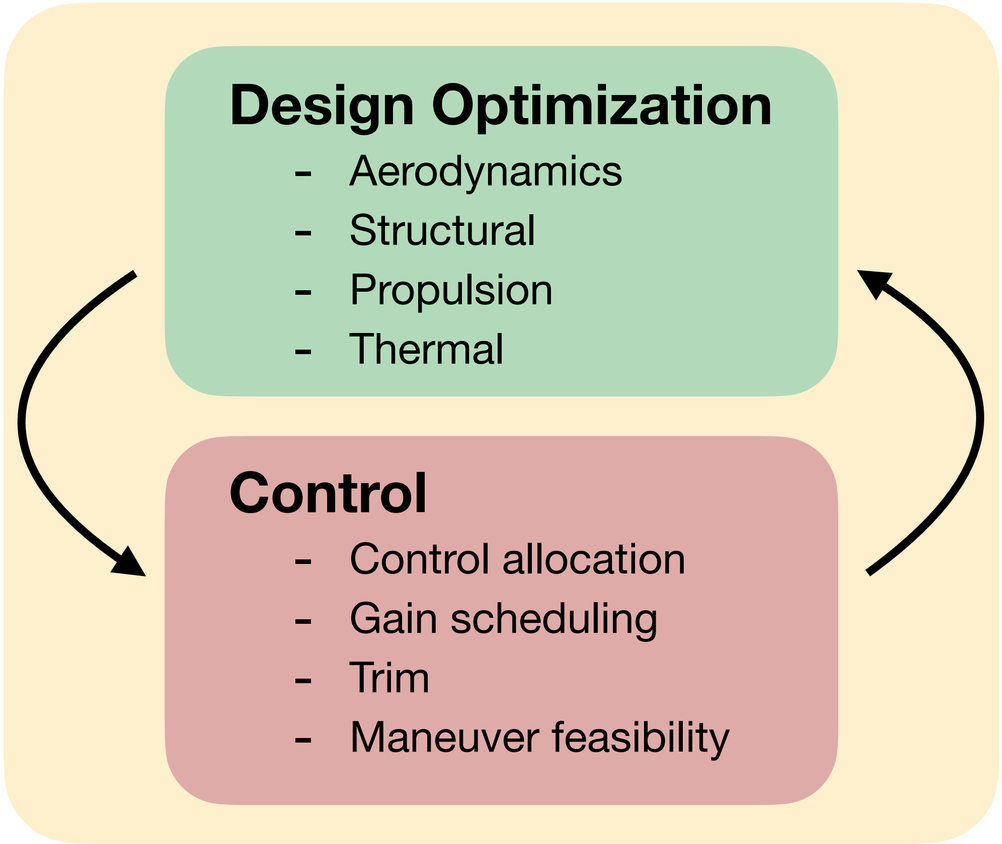

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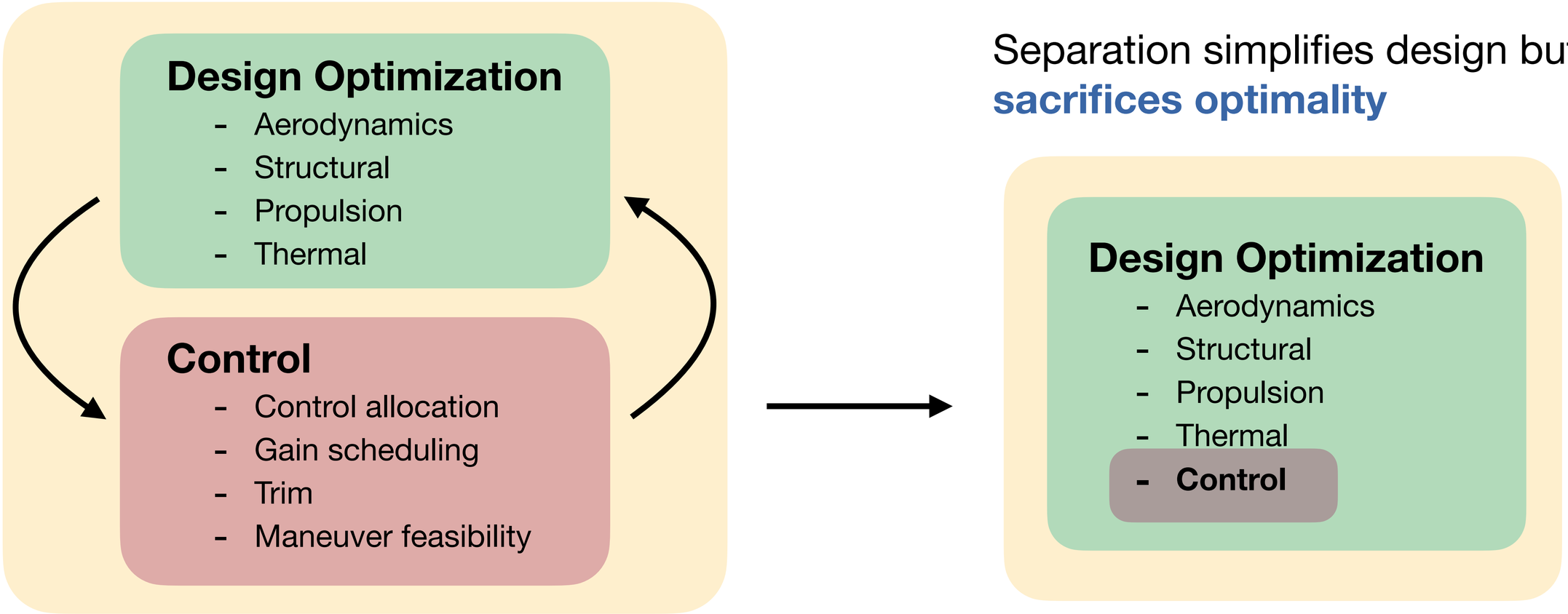

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Design Optimization and Control for Aircraft

- **Control co-design** incorporates control into earlier stages of design
- Physical limitations to control are costly to overcome
- Understanding controlled limitations can guide the design of aircraft that are **more efficient to control**

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Grumman X29

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Prior Work in Control Co-Design for Aircraft

1. Simultaneous plant and controller design
    - Tuning of LQR Q, R matrices[1]
    - Incorporating input signals as design variables[2]

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[3] R. Gupta, W. Zhao, and R. K. Kapania, “Controllability Gramian as Control Design Objective in Aircraft Structural Design Optimization,” AIAA Journal, vol. 58, no. 7, pp. 3199–3220, Jul. 2020, doi: 10.2514/1.J059102.
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   - Frequency domain results like Bode sensitivity integral
   - Controllability Gramian[3] or optimal control problems[4]

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We propose using **reachability in design optimization** to improve maneuverability

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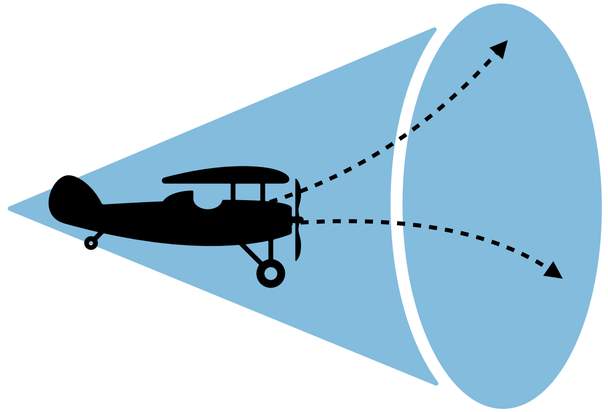
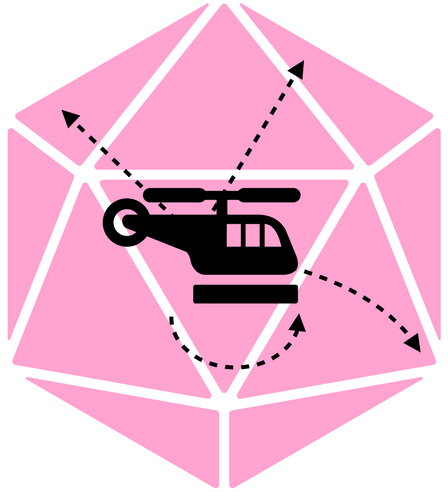
Reachability for Aircraft

- Reachability has applications for safe aircraft control[1,2]
- However, mostly unexplored in context of design optimization
  - Controllability Gramian captures $\mathscr{L}^2$ reachable set
  - Optimal control problems check inclusion in reachable set
- Aircraft design requires checking feasible maneuvers, reachability provides shortcut





[1] A. M. Bayen, I. M. Mitchell, M. M. K. Oishi, and C. J. Tomlin, “Aircraft Autolander Safety Analysis Through Optimal Control-Based Reach Set Computation,” Journal of Guidance, Control, and Dynamics, vol. 30, no. 1, pp. 68–77, Jan. 2007, doi: 10.2514/1.21562.
[2] R. Teo, J. S. Jang, and C. Tomlin, “Flight Demonstration of Provably Safe Closely Spaced Parallel Approaches,” in AIAA Guidance, Navigation, and Control Conference and Exhibit, San Francisco, California: American Institute of Aeronautics and Astronautics, Aug. 2005. doi: 10.2514/6.2005-6197.

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Reachable Sets of Interest

$$x(t) \in \mathbb{R}^n,\ u(t) \in \mathbb{R}^m,\ d \in \mathbb{R}^{n_d},$$

We consider linear-longitudinal models for aircraft dynamics (parameterized by design parameter d)

$$\dot{x} = A(d)x + B(d)u \qquad x = \begin{bmatrix} V \\ \alpha \\ Q \\ \theta \end{bmatrix}$$

- V — Velocity
- α — Angle of Attack
- Q — Pitch Rate
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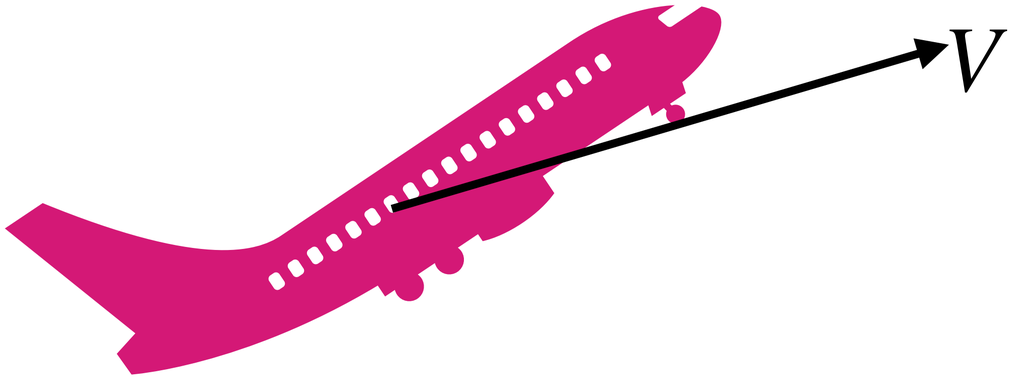

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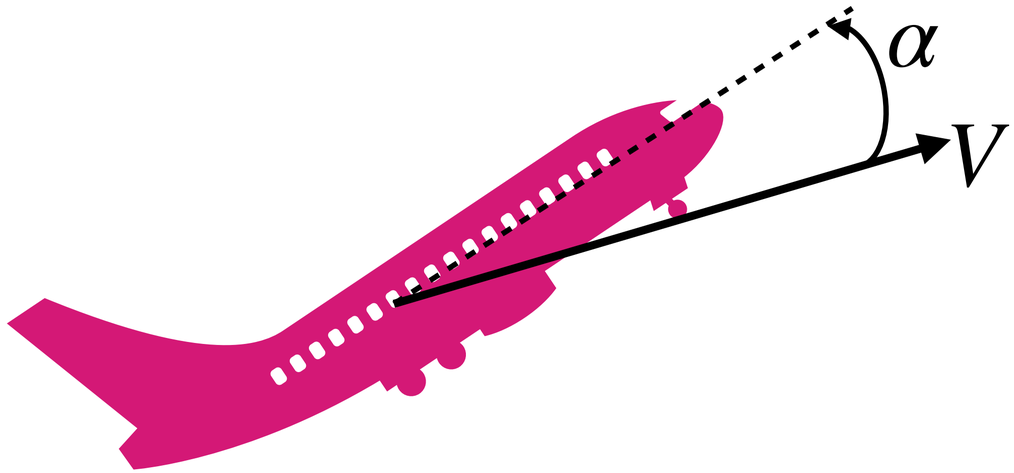

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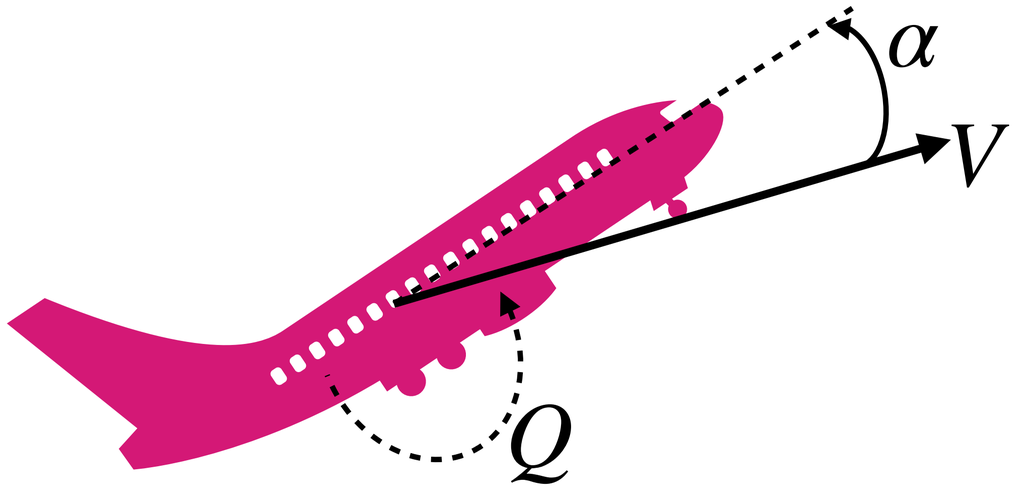

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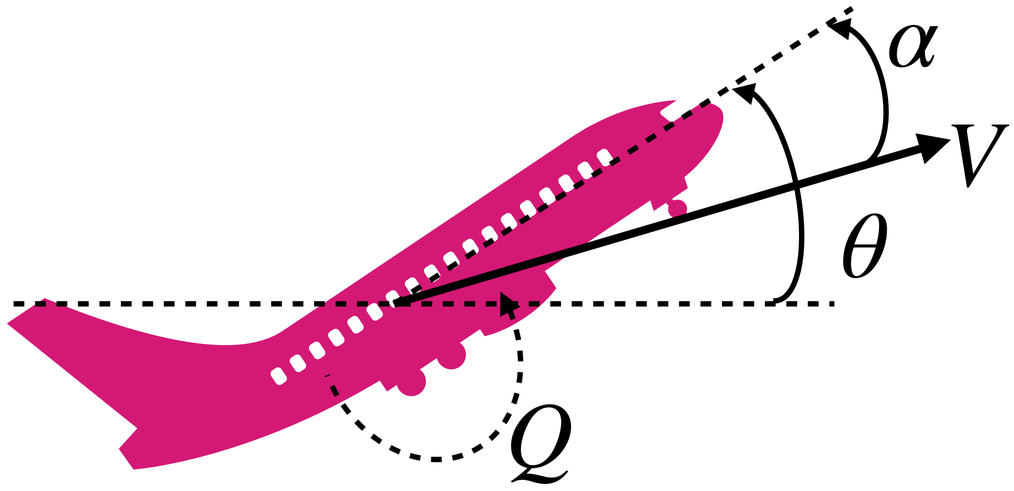

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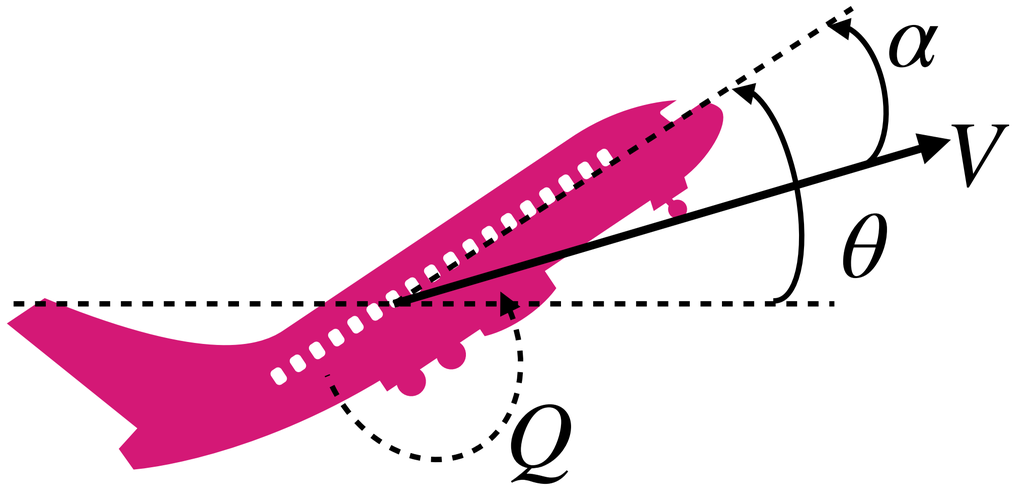

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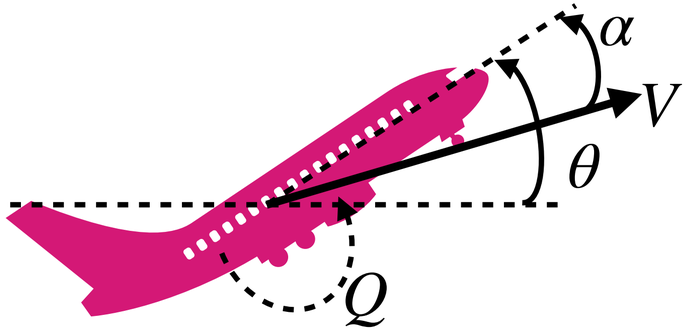

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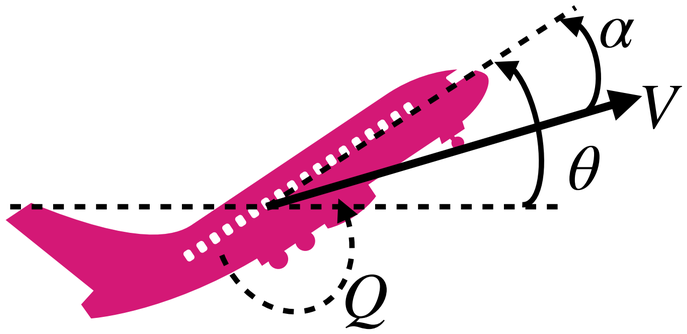
Dynamics are linearized around steady flight at $V = 200$ m/s, $\theta - \alpha = 0$ rad, $Q = 0$ rad/s



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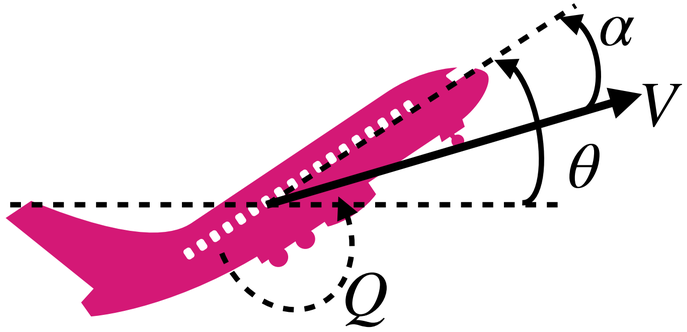

We consider reachable sets using inputs of bounded $\mathscr{L}^p$ norms, from $p = 2, \ldots, \infty$:

$$\mathscr{R}(t_f;\ d,\ p) = \{x(t_f) \,|\, \dot{x} = A(d)x + B(d)u,\ x(t_0) = 0,\ \|u(t)\|_{\mathscr{L}^p} \leq \xi\}$$

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Sampling-Based Reachability

Sampling with optimal controls
identifies boundaries of reachable set

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Sampling-Based Reachability

Sampling with optimal controls identifies boundaries of reachable set

Minimum $\mathscr{L}^2$-norm control to reach x_f at time t_f is:

$$W_R = \int_{t_0}^{t_f} e^{A(t_f-\tau)} B B^\top e^{A^\top (t_f-\tau)} d\tau$$

$$u^*(t; x_f) = B^\top e^{A^\top (t_f - t)} W_R^{-1} x_f$$

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In the general $\mathscr{L}^p$ case $(1 \leq p < \infty)$, control is:

$$u^*(t; \lambda) = \left(-B^\top e^{-A^\top t} \lambda \right)^{\circ \frac{1}{p-1}}$$

Hadamard power

costate in $\mathbb{R}^n$

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costate in $\mathbb{R}^n$

For $\mathscr{L}^\infty$, or magnitude-bounded inputs ($|u(t)| \leq \bar{u}$), the control is:

$$u_i^*(t, \lambda) = \begin{cases} \bar{u}_i, & \text{sgn}(B^\top e^{A^\top (t_f - t)} \lambda) \geq 0 \\ -\bar{u}_i, & \text{sgn}(B^\top e^{A^\top (t_f - t)} \lambda) < 0 \end{cases}$$

$$i = 1, \ldots, m$$

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Reachable Set Boundaries

$$u_i^*(t,\lambda) = \begin{cases} \overline{u}_i, & \text{sgn}(B^\top e^{A^\top(t_f-t)}\lambda) \geq 0 \\ -\overline{u}_i, & \text{sgn}(B^\top e^{A^\top(t_f-t)}\lambda) < 0 \end{cases}$$

Lemma: For every exposed point $q \in \mathscr{R}(t_f)$, there exists a λ satisfying $\|\lambda\|_2 = 1$ such that the corresponding control u^* drives the system from $x(t_0) = 0$ to $x(t_f) = q$

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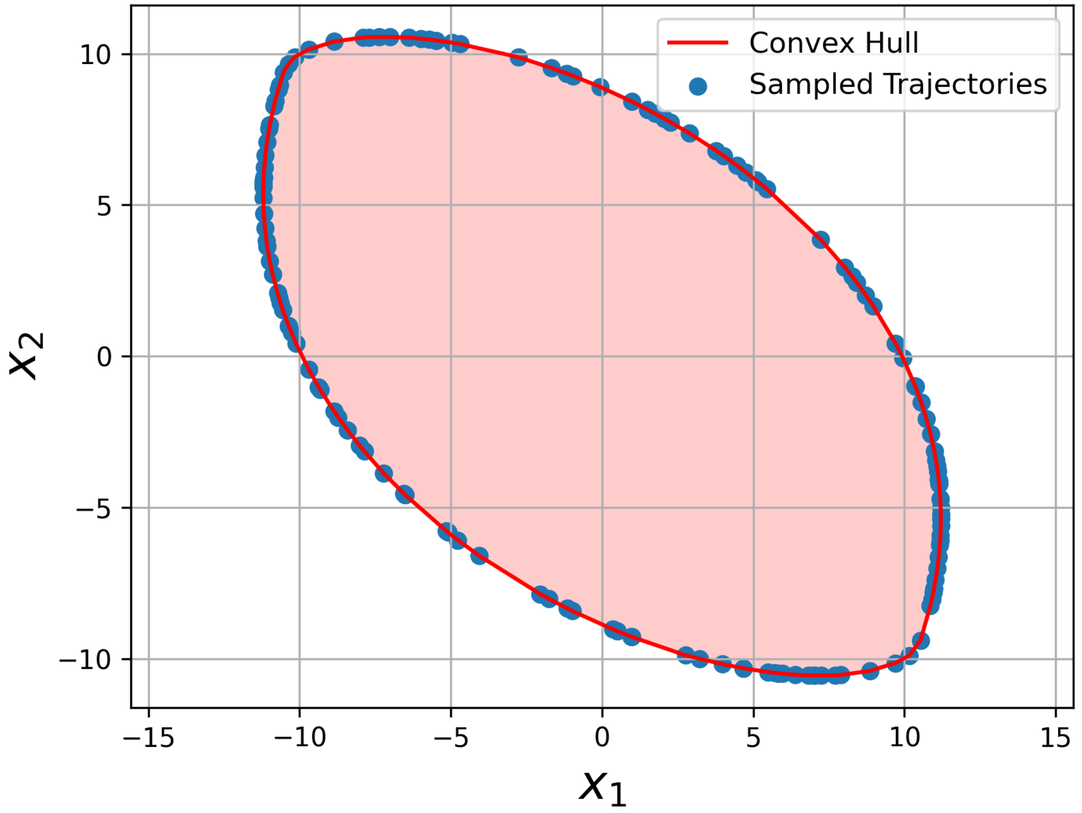
Example:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$-1 \leq u(t) \leq 1$$

Sweeping across λ on the unit sphere and sampling trajectories using $u^*(t;\lambda)$, we find the set:



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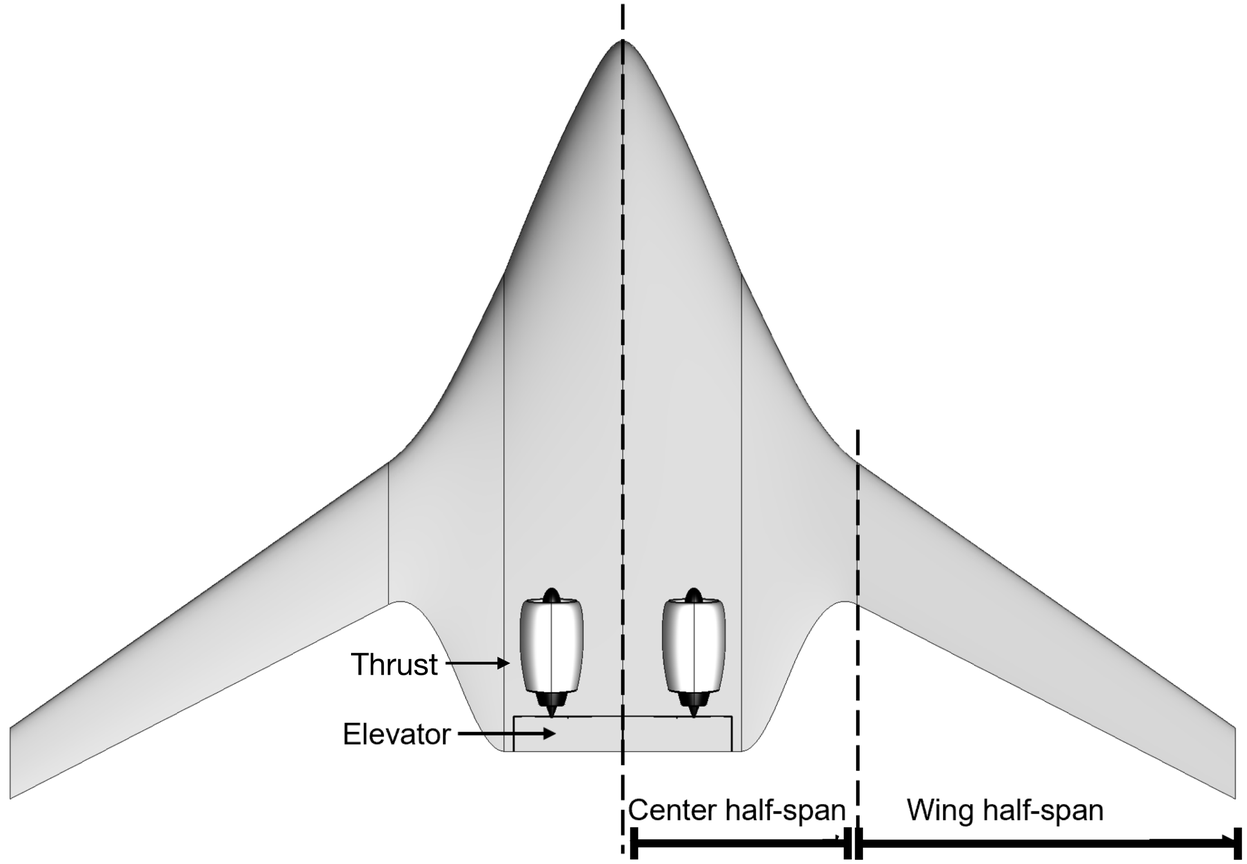
Blended-Wing-Body Application

We use reachability in design optimization of a **blended-wing-body (BWB) aircraft**

Aerodynamic analysis used to construct linearized dynamics $A(d), B(d)$

Optimize geometry using **reachable set volume and projection**



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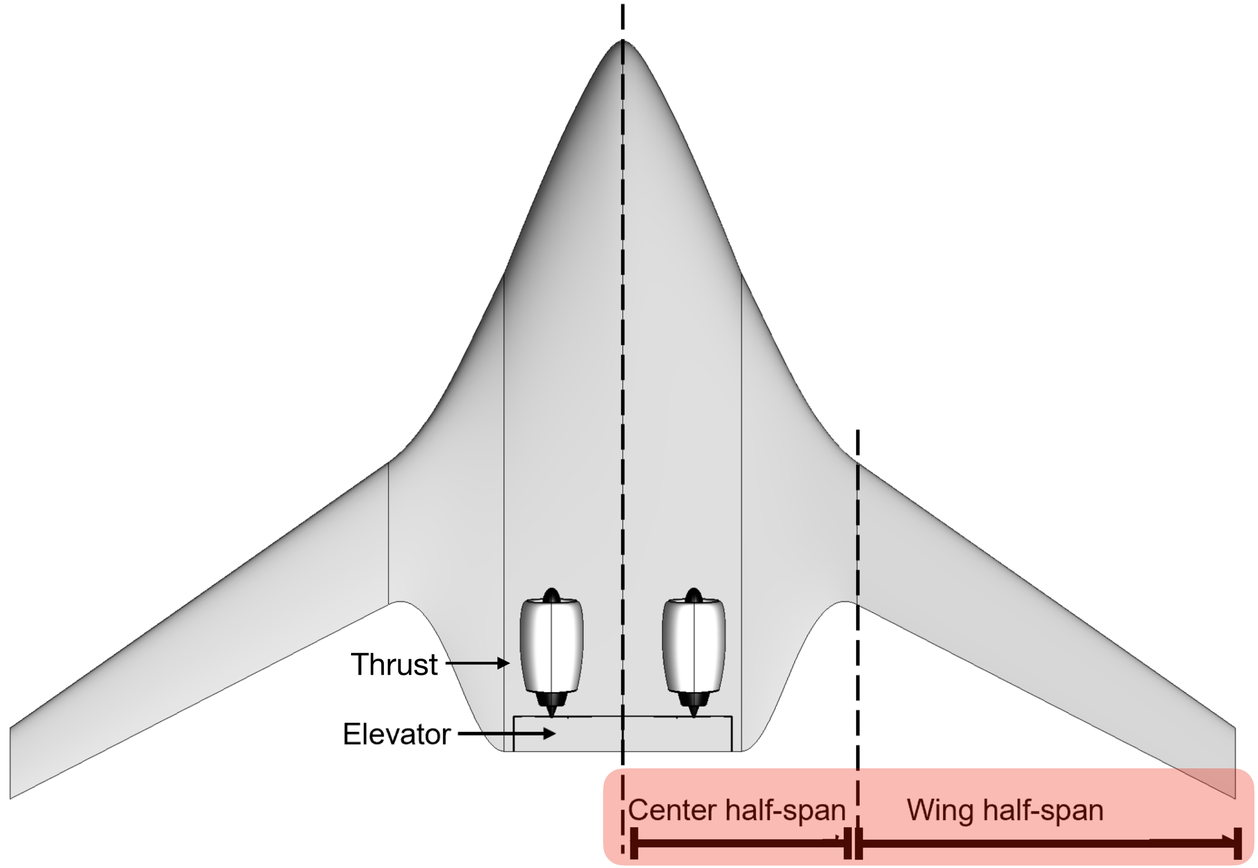

Design parameters

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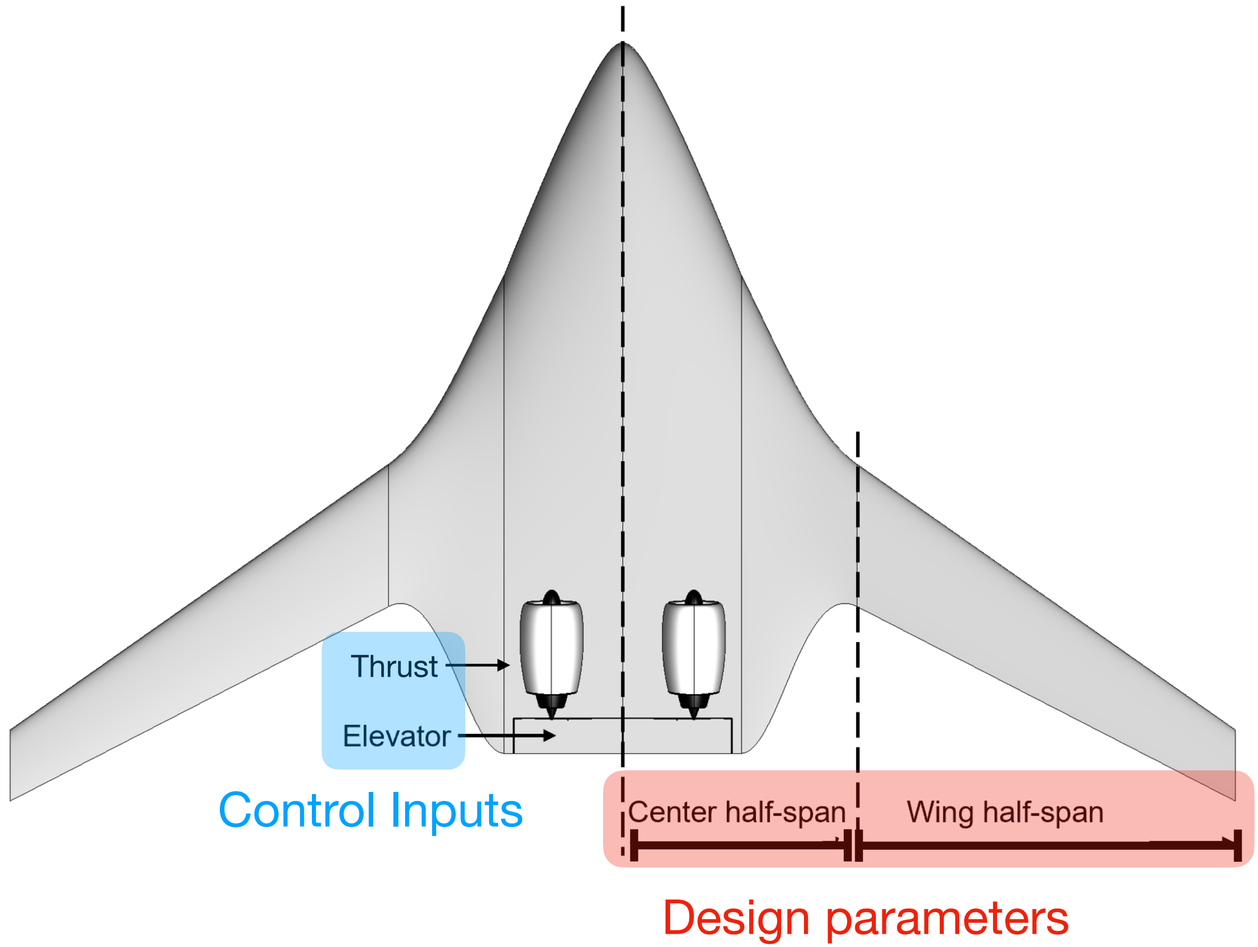

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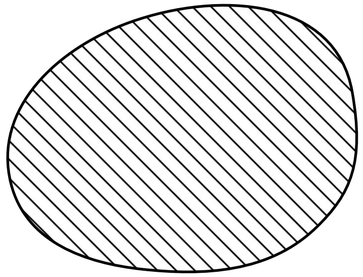
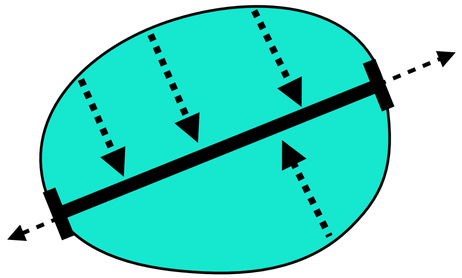
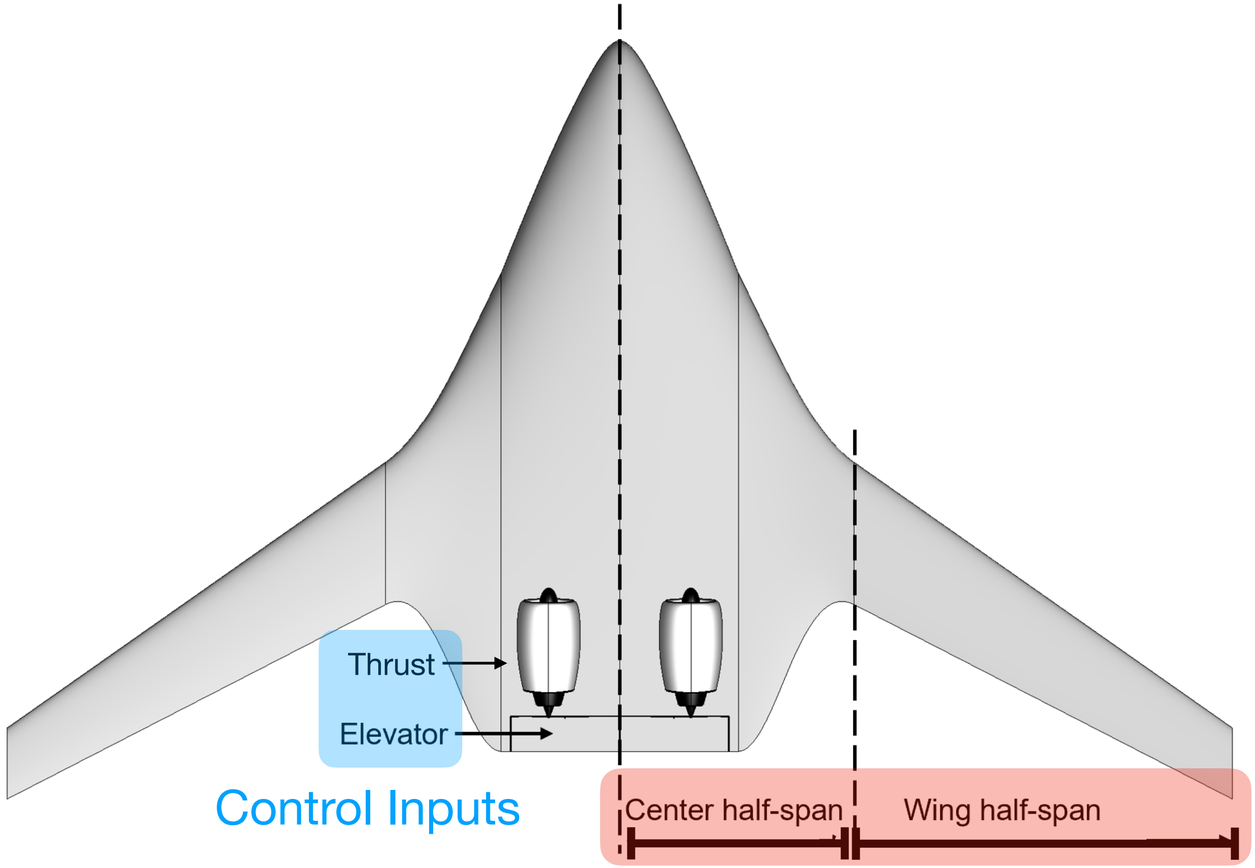


Volume



Projection



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Optimization Problems

$$\begin{aligned} \max_{c,w} \quad & \text{vol}(\mathscr{R}(t_f)) \\ s.t. \quad & c \in [3,7] \\ & w \in [10,20] \end{aligned}$$

$$\begin{aligned} \max_{c,w} \quad & \text{Proj}_{\psi}(\mathscr{R}(t_f)) \\ s.t. \quad & c \in [3,7] \\ & w \in [10,20] \end{aligned}$$

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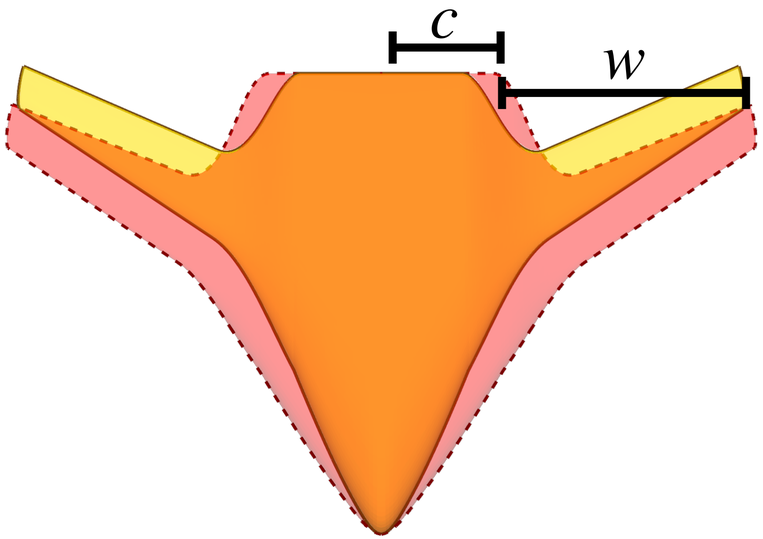
$c^* = 7$ m

$w^* = 10.5$ m



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$$w \in [10,20]$$
$$\text{Proj}_{\psi}(\mathscr{R}(t_f)) > \beta$$

$c^* = 7$ m

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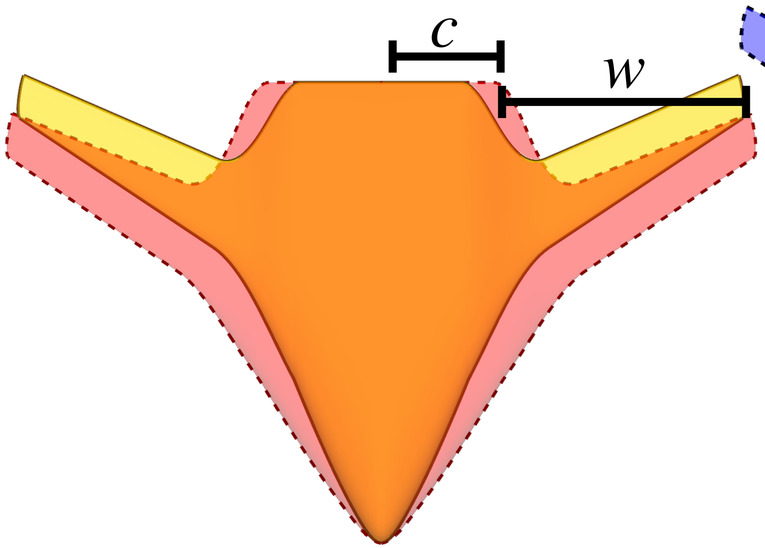
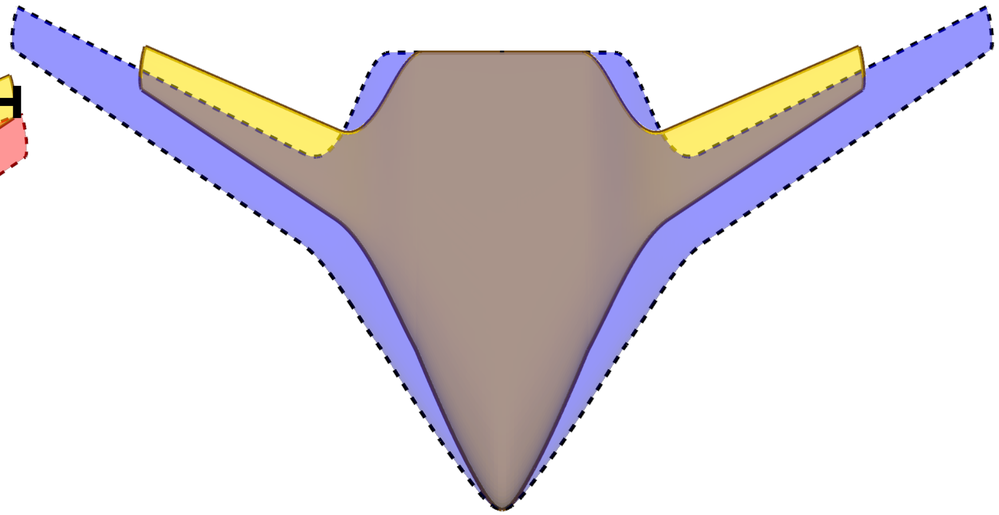


$c^* = 7$ m

$w^* = 18.0$ m



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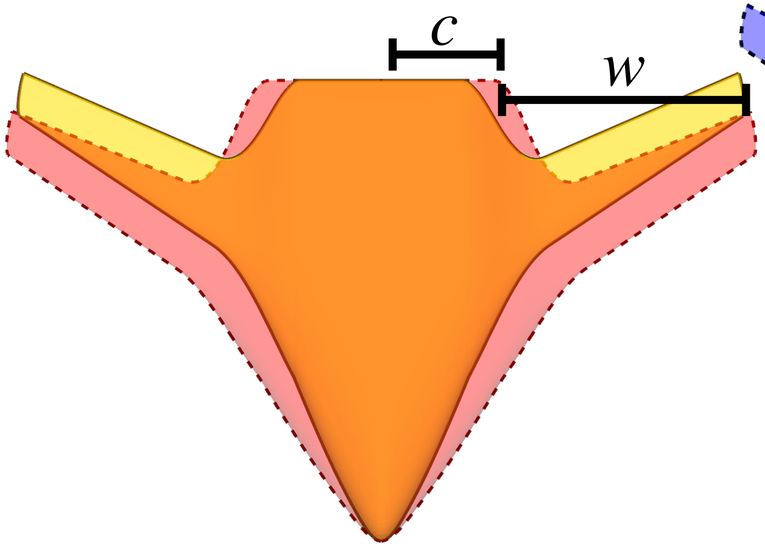

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$c^* = 7$ m

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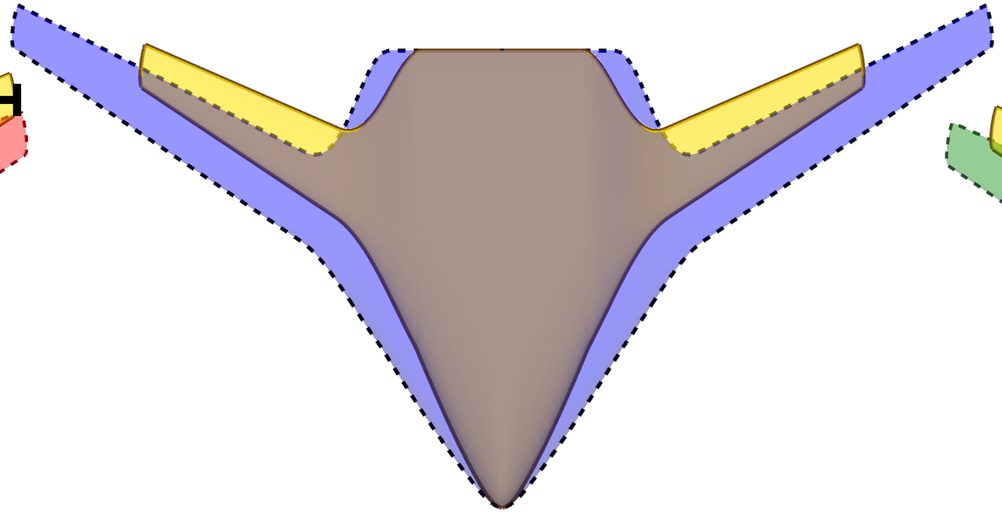
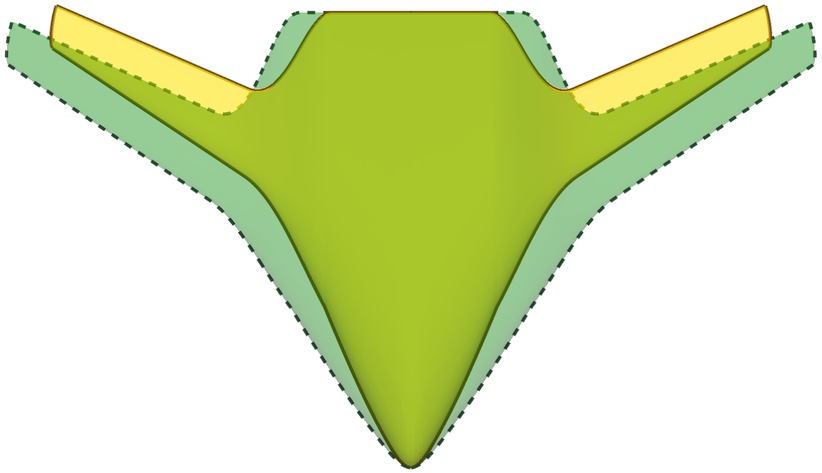


$$\max_{c,w} \quad \text{vol}(\mathscr{R}(t_f))$$
$$s.t. \quad c \in [3,7]$$
$$w \in [10,20]$$
$$\text{Proj}_{\psi}(\mathscr{R}(t_f)) > \beta$$

$c^* = 7$ m

$w^* = 12.6$ m



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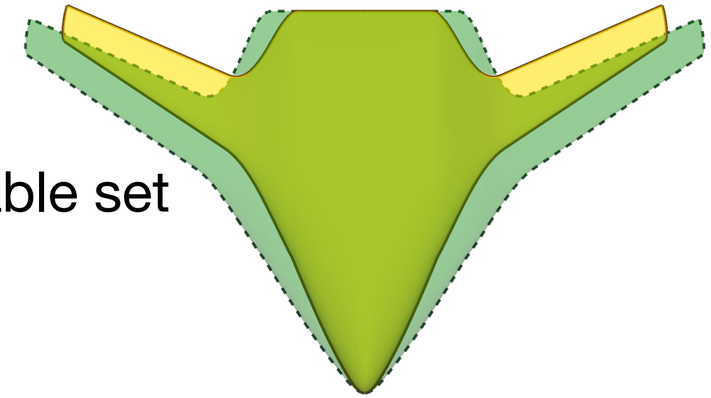
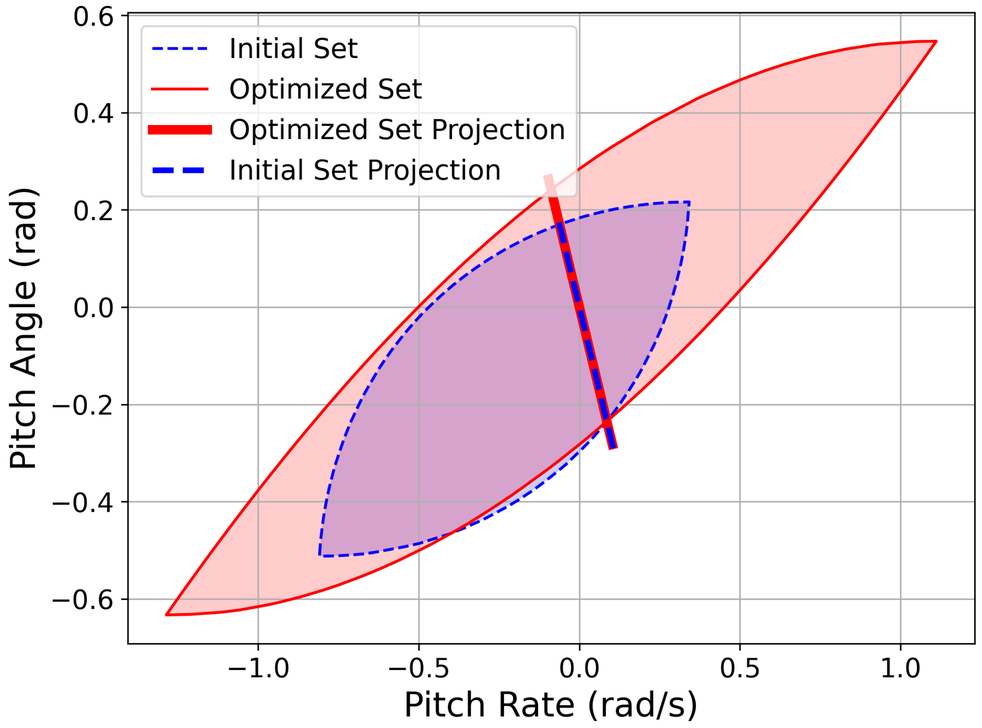
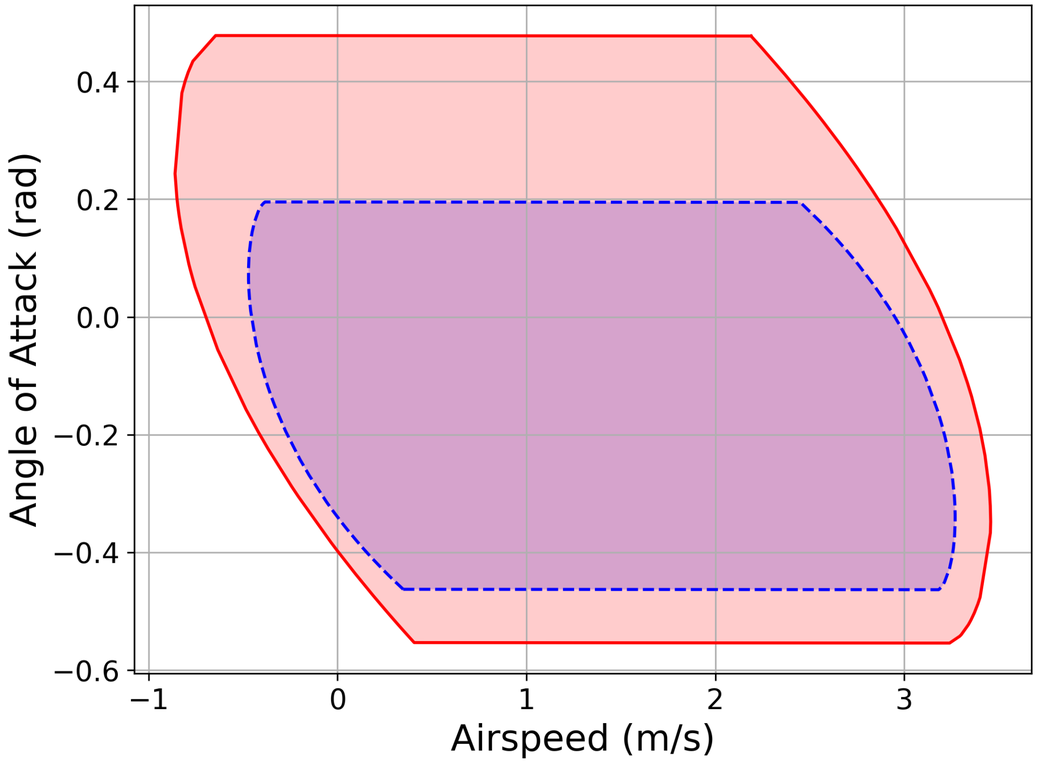
Reachable Set Visualization



Projection of 4D reachable set into 2D clearly shows a larger reachable set after optimization





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Reference Tracking Comparison

To analyze changes in controlled performance, consider reference trajectory with **two stages**:

1. **Quasi-steady climb** for 20 seconds, $V_{\text{des}} = 190$ m/s, $q = 0$ rad/s, $\gamma_{\text{des}} = 10^{\circ}$

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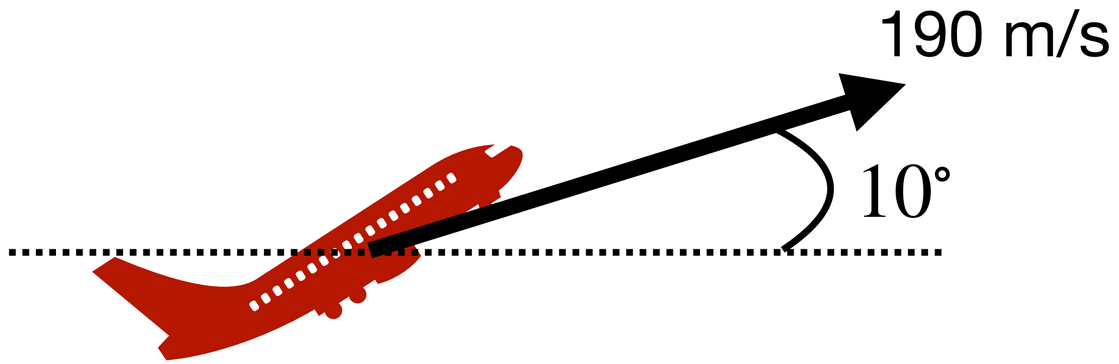

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2. **Level flight** for 20 seconds, $V_{\text{des}} = 210$ m/s, $q = 0$ rad/s, $\gamma_{\text{des}} = 0^{\circ}$

Initial condition was trim point at $V_{\text{des}} = 185$ **m/s,** $q = 0$ **rad/s,** $\gamma_{\text{des}} = 0^{\circ}$

Controller uses an integrator and LQR

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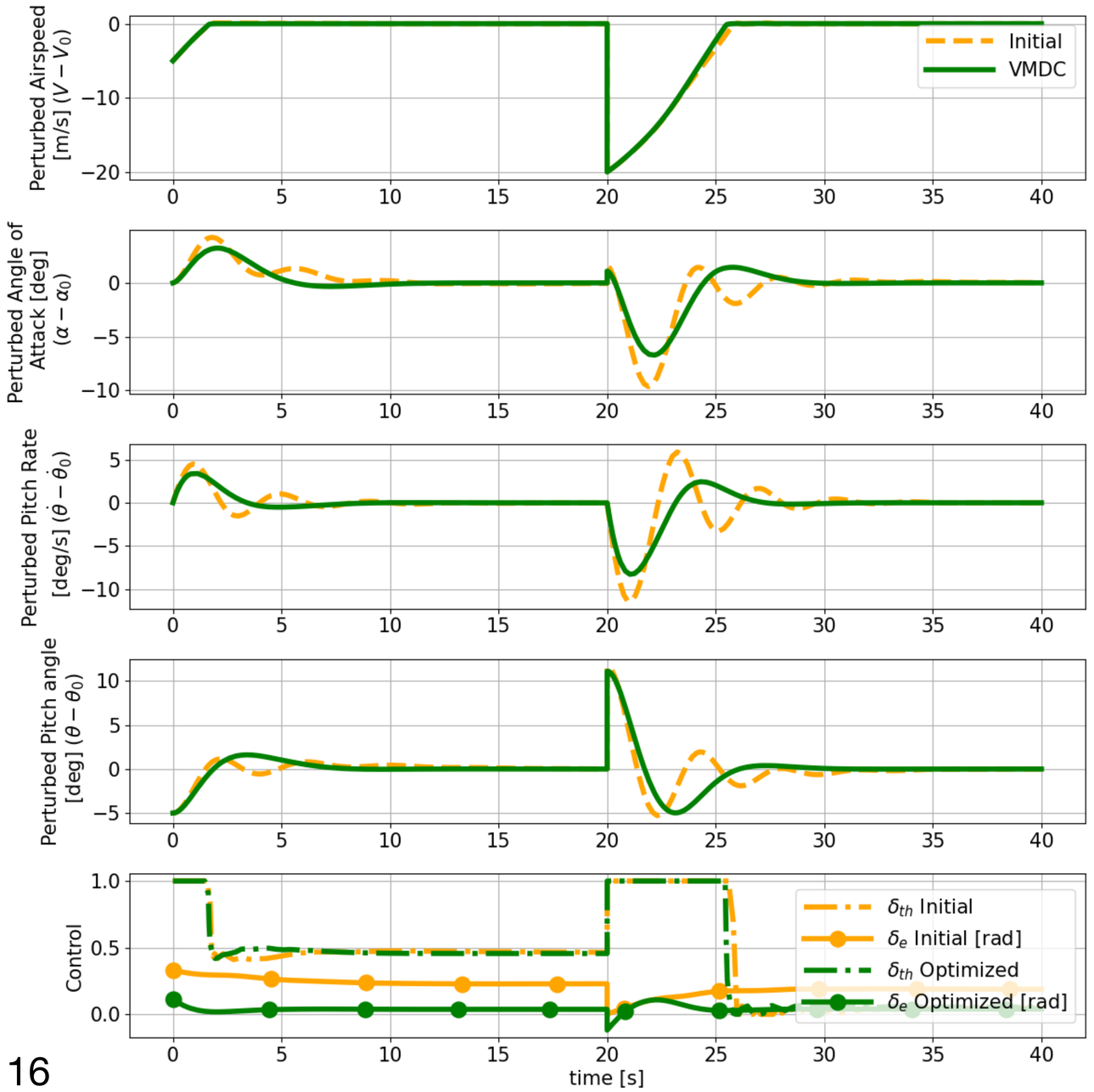
Improvements in Control

Optimized designs exhibit improved reference tracking

Plotting tracking error and control signals for initial and optimized design

Less oscillation in response of optimized model

Less elevator control effort



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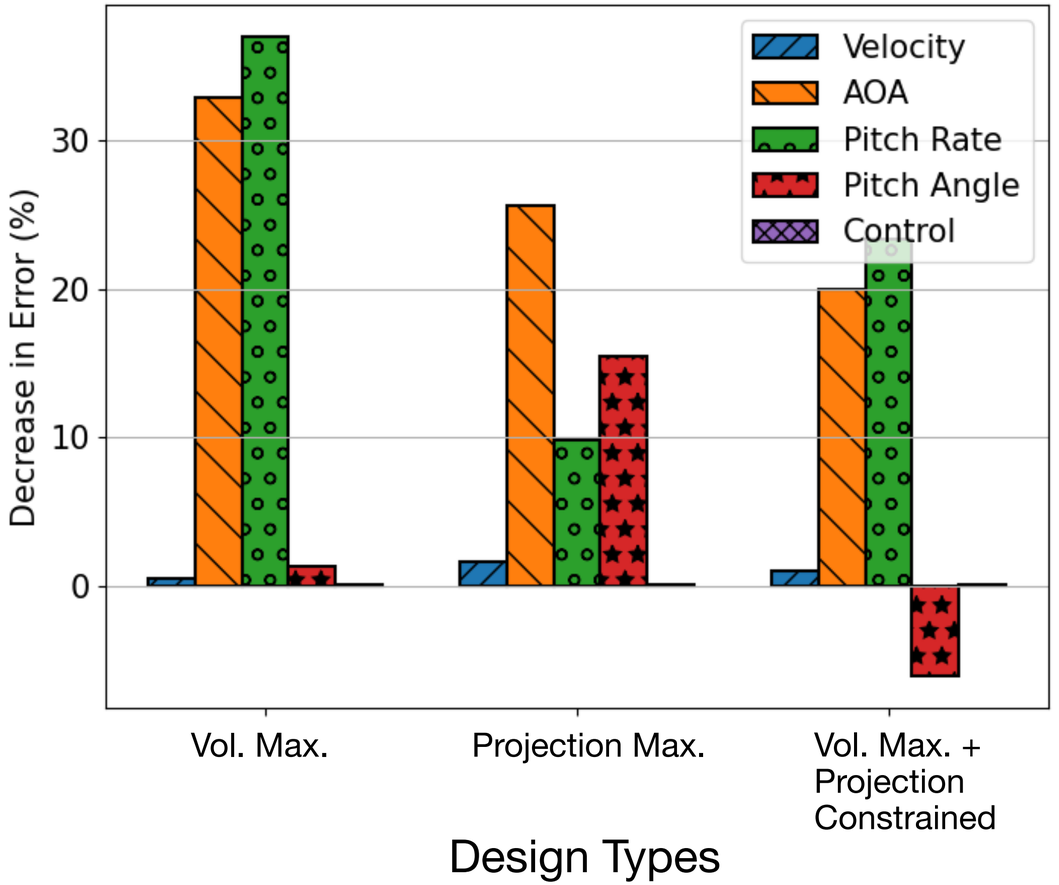
Error Comparison

$\mathscr{L}^2$ norms of tracking errors improve overall

Notable improvements in angle of attack and pitch rate

Improvements in velocity small since already easy to control in original design

Pitch angle error may be due to controller tuning



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Summary and Future Works

In summary:

1. We propose formulating design optimization problems using reachability analysis
2. Applying to blended-wing-body aircraft, we find improvements in controlled performance

Link to journal submission

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Summary and Future Works

In summary:

1. We propose formulating design optimization problems using reachability analysis
2. Applying to blended-wing-body aircraft, we find improvements in controlled performance

Future directions:

1. Reachable sets with magnitude and derivative input bounds
2. Considering lateral dynamics
3. Optimizing reachability metrics at multiple flight conditions simultaneously
4. Considering additional design variables

Link to journal submission

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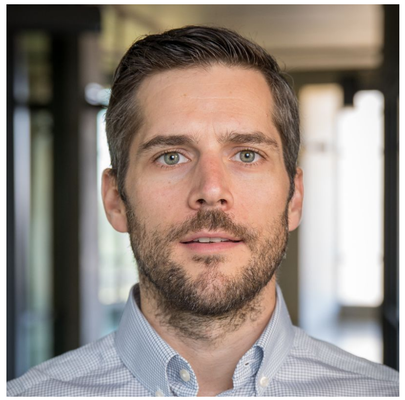
Acknowledgements

Thanks to advisors Prof. Boris Kramer and Prof. Jorge Cortés

Also thanks to Nicholas Orndorff



Link to journal submission

We would like to acknowledge the Multidisciplinary Science and Technology Center of the Aerospace Systems Directorate, Air Force Research Laboratory for funding and supporting this effort through the Collaborative Center for Design and Research of Interdisciplinary Systems program

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Extra slides

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Linear Longitudinal Dynamics

Dynamics linearized at $V = 200$ m/s, $\gamma = \theta - \alpha = 0$ rad, $Q = 0$ rad/s

$$\begin{bmatrix} \dot{V}(t) \\ \dot{\alpha}(t) \\ \dot{Q}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} X_V & X_\alpha & 0 & -g\cos\gamma_0 \\ \frac{Z_V}{V_0} & \frac{Z_\alpha}{V_0} & 1+\frac{Z_Q}{V_0} & -\frac{g\sin\gamma_0}{V_0} \\ M_V & M_\alpha & M_Q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V(t) \\ \alpha(t) \\ Q(t) \\ \theta(t) \end{bmatrix} + \begin{bmatrix} X_{\text{th}}\cos\alpha_0 & X_{\text{e}} \\ -X_{\text{th}}\sin\alpha_0 & \frac{Z_{\delta_{\text{e}}}}{V_0} \\ M_{\delta_{\text{th}}} & M_{\delta_{\text{e}}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{\text{th}}(t) \\ \delta_{\text{e}}(t) \end{bmatrix}$$

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Control Design with Integral LQR

Considering the linear dynamics (about some trim point):

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad x = \begin{bmatrix} V \\ \alpha \\ q \\ \theta \end{bmatrix}$$

Add an integrator on pitch angle that stabilizes to 0:

$$\dot{\sigma}(t) = Cx(t) - r_{\text{ref}}(t) \qquad C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{\text{ref}} = 0$$

Combined dynamics in $z(t) = \begin{bmatrix} x(t)^\top & \sigma(t) \end{bmatrix}^\top$:

$$\dot{z}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{\sigma} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t)$$

$$= \tilde{A}z(t) + \tilde{B}u(t)$$

Controller gain K solved using LQR

$$u(z(t)) = -Kz(t)$$

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Gain + Phase Margins

Model	Phase Margin [deg]	Gain Margin [dB]	Gain Cross. Freq. [rad/s]
Initial	27.7	10.2	1.48
Opt. 1	52.9	23.4	0.76
Opt. 2	36.8	9.9	1.78
Opt. 3	39.3	20.3	0.89

$$x = \begin{bmatrix} V \\ \alpha \\ Q \\ \theta \\ \sigma \end{bmatrix} \qquad u = \begin{bmatrix} \delta_{\text{th}} \\ \delta_{\text{e}} \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 1 & & & & & \\ & 100 & & & & \\ & & 1 & & & \\ & & & 100 & & \\ & & & & 100 & \end{bmatrix} \qquad R_1 = \begin{bmatrix} 0.1 & \\ & 1000 \end{bmatrix}$$

Gain + Phase Margins

Model	Phase Margin [deg]	Gain Margin [dB]	Gain Cross. Freq. [rad/s]	Phase Margin [deg]	Gain Margin [dB]	Gain Cross. Freq. [rad/s]
Initial	27.7	10.2	1.48	46.6	13.5	2.8
Opt. 1	52.9	23.4	0.76	50.2	14.3	2.86
Opt. 2	36.8	9.9	1.78	44.8	12.7	3.14
Opt. 3	39.3	20.3	0.89	48.4	13.8	2.9

$$x = \begin{bmatrix} V \\ \alpha \\ Q \\ \theta \\ \sigma \end{bmatrix} \qquad u = \begin{bmatrix} \delta_{\text{th}} \\ \delta_{\text{e}} \end{bmatrix}$$

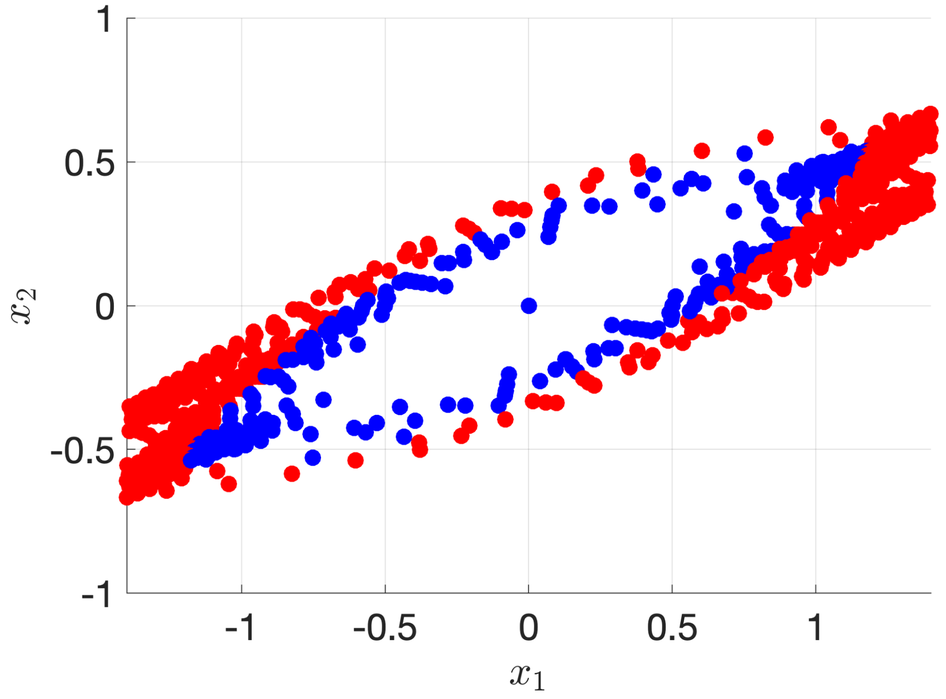
$$Q_1 = \begin{bmatrix} 1 & & & & \\ & 100 & & & \\ & & 1 & & \\ & & & 100 & \\ & & & & 100 \end{bmatrix} \qquad R_1 = \begin{bmatrix} 0.1 & \\ & 1000 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 100 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1000 & \\ & & & & 10 \end{bmatrix} \qquad R_2 = \begin{bmatrix} 0.1 & \\ & 10 \end{bmatrix}$$

Numerical Approx. of $\mathscr{L}^6$ Reachable Set

$$\dot{x} = \begin{bmatrix} 0.4 & -0.3 \\ 0.5 & 1.7 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

- Blue points **are** reachable with inputs $\|u(t)\|_{\mathscr{L}^6} \leq 1$
- Red points **not** reachable with inputs $\|u(t)\|_{\mathscr{L}^6} \leq 1$



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Optimization Using $\mathscr{L}^6$

Aero data from wind-tunnel testing for highly-maneuverable aircraft

Minimizing wing perimeter while increasing reachable set volume

Wingspan

Wing chord

$$\min_{b,\bar{c}} \quad b + \bar{c}$$

$$s.t. \quad b \in [4.6,\ 13.7]$$

$$\bar{c} \in [1.7,\ 5.2]$$

$$\text{vol}(\mathscr{R}(t_f; [b,\ c],\ \mathscr{L}^6)) > 1.1\text{vol}(\mathscr{R}(t_f; [b_0, \bar{c}_0],\ \mathscr{L}^6))$$

$$[b_0,\ \bar{c}_0] = [9.14,\ 3.45] \longrightarrow [b^*,\ \bar{c}^*] = [13.6,\ 3.2]$$

[1] L. T. Nguyen, Marilyn E. Ogburn, William P. Gilbert, Kemper S. Kibler, Phillip W. Brown, and Perry L. Deal, Simulator study of stall/post-stall characteristics of a fighter airplane with relaxed longitudinal static stability. in NASA langley research center report. National Aeronautics and Space Administration, 1979.

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